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Elastic collision in one dimension viewed as a linear transformation

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The solution for a two-body elastic collision in one dimension is well known. Let u_1, u_2 be the initial velocities and v_1, v_2 be the velocities after collision, where the indices 1 and 2 refer to the particle with mass m_1 and m_2, respectively (see Fig. 1). The final velocities can be solved in terms of the initial velocities (see, for example, Ref. 1, p. 217)

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1 + \frac{2m_2}{m_1 + m_2} u_2, \tag{1}$$

$$v_2 = \frac{2m_1}{m_1 + m_2} u_1 + \frac{m_2 - m_1}{m_2 + m_2} u_2. \tag{2}$$

Now let us view the problem as a linear transformation. Define the velocity vectors

$$\mathbf{u} = (u_1, u_2), \quad \mathbf{v} = (v_1, v_2).$$

Then (1) and (2) can be written compactly as

$$\mathbf{v} = M\mathbf{u}, \tag{3}$$

where

$$M = \begin{bmatrix} \dfrac{m_1 - m_2}{m_1 + m_2} & \dfrac{2m_2}{m_1 + m_2} \\[2ex] \dfrac{2m_1}{m_1 + m_2} & -\dfrac{m_1 - m_2}{m_1 + m_2} \end{bmatrix}.$$

The matrix M has a number of interesting properties: notably, (a) M is its own inverse, i.e.,

$$MM = I;$$

(b) M has two real eigenvalues $\lambda = \pm 1$ obtained by solving the equation

$$\det |M - \lambda I| = 0.$$

Both (a) and (b) will be shown to have simple meanings.

Property (a) expresses the fact that Newton's laws of motion are invariant under time reversal. Multiply (3) on both sides by M. We have

$$M\mathbf{v} = MM\mathbf{u} = \mathbf{u};$$

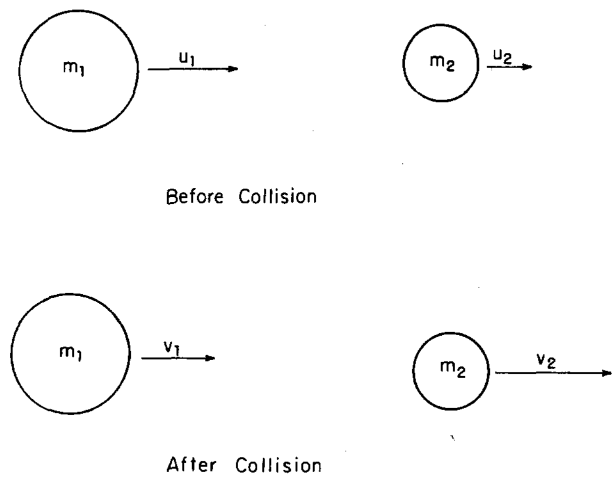

Fig. 1. Two particles before and after an elastic collision.

therefore,

$$(-\mathbf{u}) = M(-\mathbf{v}). \tag{4}$$

If we take a movie of the collision described by (3) and run it backwards, the observer would observe a collision in which the initial velocities are $-(v_1, v_2)$ and final velocities $-(u_1, u_2)$. This is exactly the content of (4).

Property (b) suggests that M can be transformed into a diagonal matrix by a suitable similarity transformation

$$M^* = SMS^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \tag{5}$$

where

$$S = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}$$

is a nonsingular matrix. It is straightforward to show that (5) imposes two constraints on s_{ij}

$$m_2 s_{11} - m_1 s_{12} = 0, \tag{6}$$

$$s_{21} + s_{22} = 0. \tag{7}$$

These two equations will determine the matrix S up to two arbitrary constants. Let

$$s_{11} = [m_1/(m_1 + m_2)]\alpha, \quad s_{22} = -\beta,$$

where α and β are two arbitrary nonzero constants. Therefore,

$$S=\begin{bmatrix}[m_1/(m_1+m_2)]\alpha & [m_2/(m_1+m_2)]\alpha \\ \beta & -\beta\end{bmatrix}. \tag{8}$$

Define

$$\mathbf{u}^* = S\mathbf{u}, \tag{9}$$

$$\mathbf{v}^* = S\mathbf{v}. \tag{10}$$

Substitution into (3) leads to

$$\mathbf{v}^* = M^*\mathbf{u}^*,$$

where M^* is the diagonal matrix in (5). This is (3) written in "normal" coordinates. The eigenvector $\mathbf{u}^*$ has components

$$u_1^* = \alpha\frac{m_1u_1+m_2u_2}{m_1+m_2},$$

$$u_2^* = \beta(u_1-u_2).$$

Up to a multiplicative constant, u_1^* and u_2^* are the familiar center-of-mass velocity and the relative velocity, respectively.

[1]R. Resnick and D. Halliday, *Physics* (Wiley, New York, 1966), part 1.

Development of educational materials to recruit women into scientific careers*

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There is no evidence that women are intellectually inferior to men either in logical reasoning or mechanical problems. Yet a disproportionate number of qualified women do not choose to pursue careers in science.

This situation is deplorable on two counts: (i) It represents an unfortunate loss of talent and potential to a profession that requires unique abilities. (ii) It narrows the effective range of career choices for women by eliminating a career that provides unusually rewarding work.

The widespread and persistent pattern of sex-role stereotyping imposed on young people by family, educators, and other social forces all have an impact on women's perception of career opportunities for themselves.[1]

Science is perceived as "masculine" by young women who see it in conflict with their emerging "femininity." Zinberg notes, "In most books written about successful scientists, even those by women, the only reference to women in the index is likely to be under 'wives of'."[2]

Preliminary work showed that many instructors and career counselors would emphasize the roles of women in science more than they do at present if suitable materials for such presentations were conveniently available.

The purpose of this research project was twofold: (i) to develop a multimedia packet, *Women in Science*, containing cassette interviews, slides, and articles plus references, showing the work and lifestyles of six successful attractive contemporary female scientists; and (ii) to evaluate the effectiveness of the materials developed in this project to improve the attitudes of students toward careers for females in science.

In selecting the role models for the project, my goal was to make the materials useful to the largest possible number of different kinds of young people. The table of contents of *Women in Science* appears in Table I.

The role models were specifically chosen to include a variety of personal backgrounds, race, appearance, personal lifestyles, geographical location, type of employer, age and stage of career achievement. They were selected to illustrate six different fields where a young woman can realistically plan a career if she gets an adequate scientific education. The scientific work of each person is described accurately and in an interesting manner so that the materials could be used to supplement current texts, or to enliven the classroom presentation of subject matter science.

The short-term trial evaluation of the effectiveness of the multimedia materials developed in this project was conducted in consultation with Dr. Edward G. Cohen, Di-

Table I. Table of contents of *Women in Science*.

Astronomy—Dr. Virginia L. Trimble
Astronomer, University of California, Irvine; University of Maryland
Biophysics—Dr. Diana McSherry
Research biophysicist, Executive Vice-President Digisonics Corp.
Engineering—Mrs. Gwendolyn Albert
Environmental engineer, U.S. Army Corps of Engineers
Nuclear Physics—Dr. Chien-Shiung Wu
Professor of Physics, Columbia University; President, American Physical Society
Science and Society—Dr. Betsy Ancker-Johnson
Assistant Secretary of Science and Technology, U.S. Department of Commerce
Space–Life Science—Dr. Carolyn Leach
Space–life scientist, Head of Endocrine Laboratory, NASA–Johnson Space Center